

Stator, Rotor and Net fields

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1 Torque expression in terms of various fields

Look at figure 1(a) to (e), where positions of Stator field $\bar{B}_s$, rotor field $\bar{B}_r$ and the resultant (net field) in space are shown. All the fields are sinusoidally distributed.

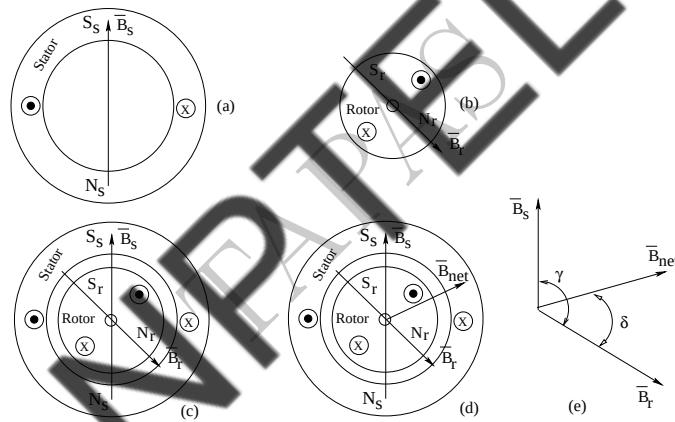


Figure 1: Stator, rotor & resultant fields.

Let the rotor coil sides carry current I_R in the directions shown in figure 2

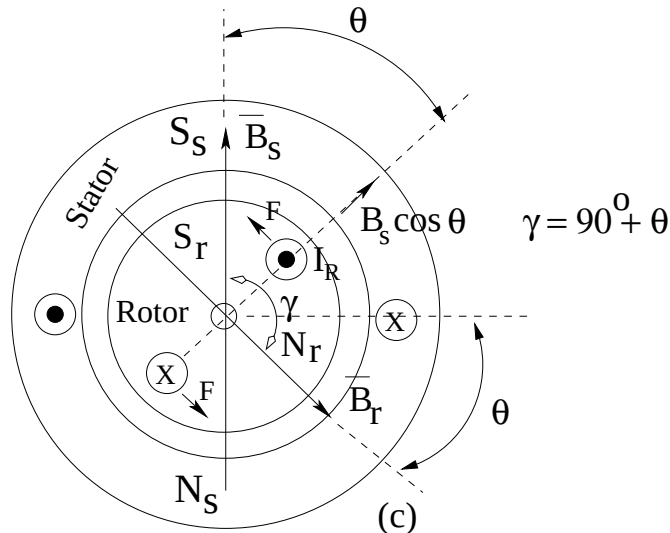


Figure 2: Stator, rotor & resultant fields.

The strength of stator field at the coil side (of length L) carrying dot current is $B_s \cos \theta$. Then by applying left hand rule, the force on it is $F = B_s \cos \theta \times I_R L$ in the direction as shown. Similarly the force on the coil side carrying cross current is $F = B_s \cos \theta \times I_R L$ but acting in the opposite direction as experienced by the other coil side. If D is the diameter of rotor, then torque acting on the rotor is given by:

$$\begin{aligned} T &= FD \\ &= B_s \cos \theta \times I_R L D \end{aligned} \quad (1)$$

But B_R is proportional to I_R and length of the coil side L and diameter of the rotor D being constant, torque can be written as:

$$\begin{aligned} T &= k B_r B_s \cos \theta \text{ where } k = \text{constant} \\ &= k B_r B_s \cos(\gamma - 90^\circ) = K B_r B_s \sin \gamma \end{aligned}$$

This can be written in terms of vector as

$$T = \bar{B}_r \times \bar{B}_s$$

In a rotating machine, stator winding will produce a stator field $\bar{B}_s$ and the rotor winding will produce a rotor field $\bar{B}_r$. If the spatial angle between the two is γ then torque experienced by the rotor will be

$$T_e = k \bar{B}_r \times \bar{B}_s = k B_r B_s \sin \gamma$$

The resultant or the net field is obviously $\bar{B}_{\text{net}} = \bar{B}_s + \bar{B}_r$ and this is important because it will determine the induced voltages in the stator and rotor coils. It can be shown that the torque can also be expressed in terms of the rotor field and the net field as follows:

$$\begin{aligned} T_e &= k \bar{B}_r \times \bar{B}_s = k B_r B_s \sin \gamma \\ \text{or, } T_e &= k \bar{B}_r \times (\bar{B}_{\text{net}} - \bar{B}_r) = k B_r B_{\text{net}} \sin \delta \end{aligned}$$

Note also that if we like the torque also can be expressed in terms of stator field and net field as follows:

$$\begin{aligned} T_e &= k \bar{B}_r \times \bar{B}_s = k B_r B_s \sin \gamma \\ \text{or, } T_e &= k (\bar{B}_{\text{net}} - \bar{B}_s) \times \bar{B}_s = k B_{\text{net}} B_s \sin \beta \end{aligned}$$

Any of the above equations can be used to obtain torque. However, the widely used one is :

$$T_e = k B_r B_{\text{net}} \sin \delta$$

The direction of the torque will be from $\bar{B}_r$ to $\bar{B}_{\text{net}}$ (right hand rule of the vector product). In case of motor operation, we know the direction of torque will be in the same direction of rotation. Therefore $\bar{B}_r$ will be always behind $\bar{B}_{\text{net}}$. On the other hand $\bar{B}_r$ will be ahead of $\bar{B}_{\text{net}}$ since the torque developed by the generator acts in the opposite direction of rotation.

2 Synchronous Machine

In this section we shall try to find out the positions of these fields for synchronous and induction machines.

2.1 Motor mode

Consider a synchronous motor operating at lagging power factor and it is rotating in the anti-clockwise direction with speed n_s as shown in figure 3(a). We first draw $\vec{B}_{\text{net}}$ in the vertical direction. Position of $\vec{B}_r$ will be behind $\vec{B}_{\text{net}}$ as shown. At this instant the stator coil sides which will have maximum induced voltage are also indicated with polarity. However this coil sides will not carry maximum current since the power factor is lagging. In fact it will carry maximum current after some time when $\vec{B}_{\text{net}}$ has advanced. Therefore the coil sides carrying maximum current must line to the right of $\vec{B}_{\text{net}}$ as shown. The dot and cross are used to indicate the direction of the maximum current.

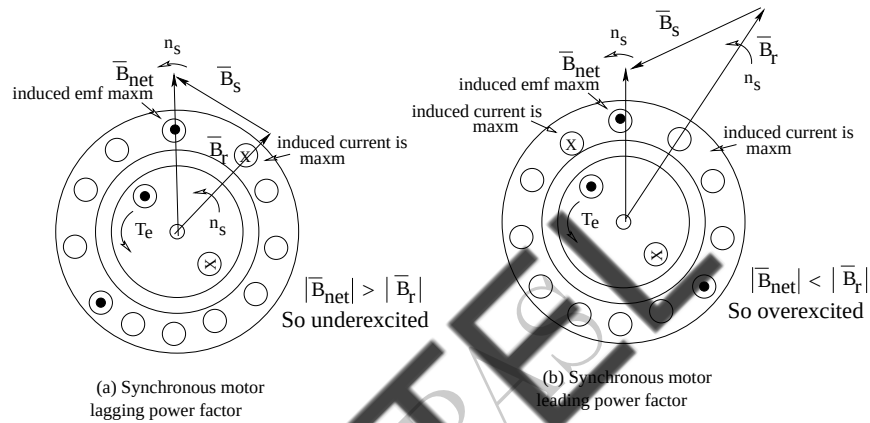


Figure 3: Synchronous motor

Similar arguments can be used to draw the field vectors shown in figure 3(b) when the motor operates at leading power factor.

2.2 Generator mode

In generator mode of operation, $\vec{B}_r$ will be ahead of $\vec{B}_{\text{net}}$ since the torque developed by the generator acts in the opposite direction of rotation. In figure 4, generator is assumed to be rotating in the **clockwise direction**. Once again all the field positions at a given instant are drawn and shown in figure 4(a) and (b) respectively for leading and lagging power factors.

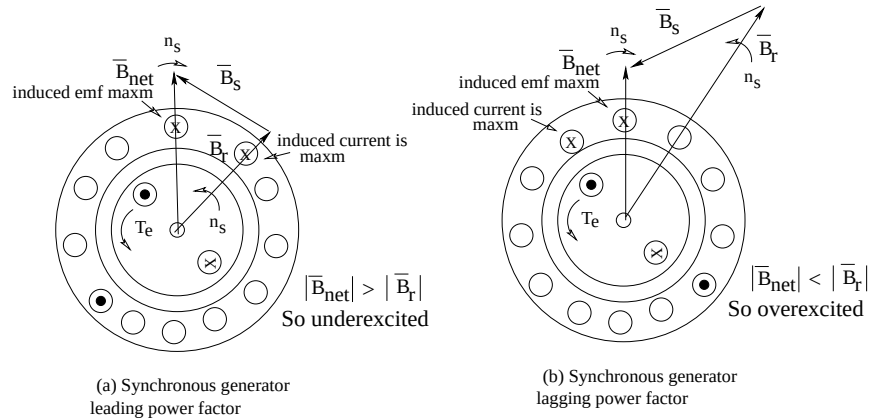


Figure 4: Synchronous generator

3 Induction machine

The relative positions of the three fields namely $\bar{B}_{\text{net}}$, $\bar{B}_s$ and $\bar{B}_r$ can also be shown for a 3-phase induction motor as well both for motor and generator mode of operation.

3.1 Induction motor

Let us assume that the speed of the fields with respect to a stationary observer be n_s and they are rotating in the anti-clockwise direction as shown in figure 5(a). Rotor speed is also assumed to be n_r along the same direction with the condition $n_r < n_s$ for motor operation. We first draw $\bar{B}_{\text{net}}$ along the vertical direction and we know that $\bar{B}_r$ will be behind $\bar{B}_{\text{net}}$ for motor operation. Now identify Coil sides which will have maximum induced voltage i.e., the coil sides underneath $\bar{B}_{\text{net}}$. Applying right hand rule the polarity of these voltages (dot or cross) are identified. Since both stator and rotor have lagging power factors, the coil sides which will have maximum current will be different. These are then identified and then positions of $\bar{B}_s$ and $\bar{B}_r$ are positioned so that $\bar{B}_{\text{net}} = \bar{B}_s + \bar{B}_r$ as shown in figure 5(a).

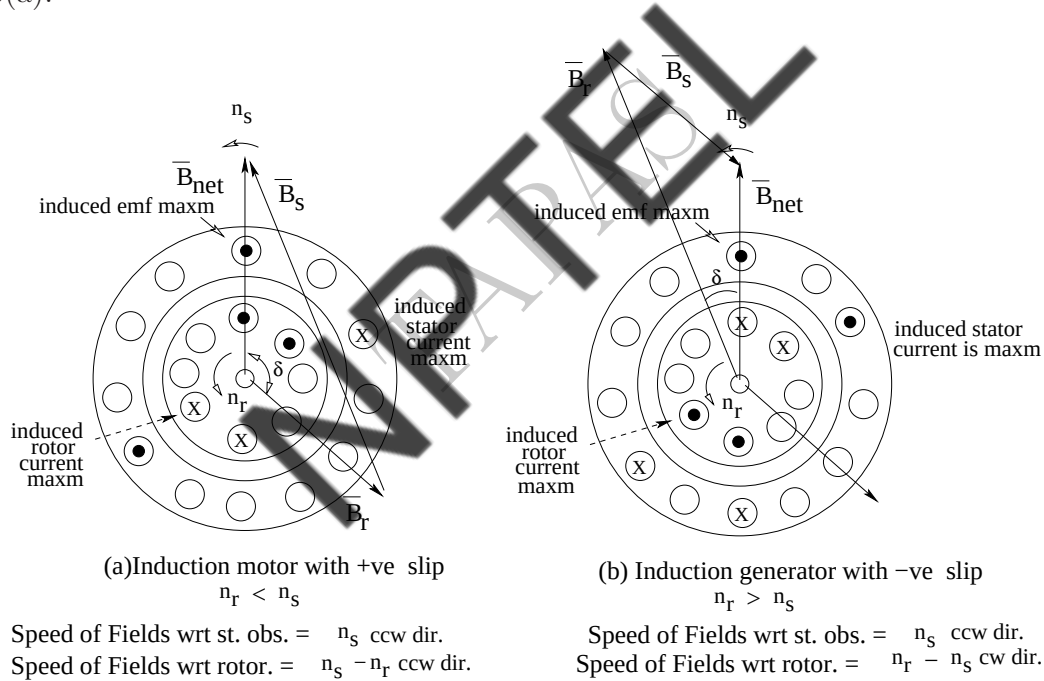


Figure 5: 3-phase induction motor

3.2 Induction generator

Let us assume that the speed of the fields with respect to a stationary observer be n_s and they are rotating in the anti-clockwise (ccw) direction as shown in figure 5(b). Rotor speed is also assumed to be n_r along the same direction with the condition $n_r > n_s$ for generator operation. We first draw $\bar{B}_{\text{net}}$ along the vertical direction and we know that $\bar{B}_r$ will be ahead of $\bar{B}_{\text{net}}$ for generator operation. Now identify Coil sides which will have maximum induced voltage i.e., the coil sides underneath $\bar{B}_{\text{net}}$. Applying right hand rule the polarity of these voltages (dot or cross) are identified. Since both stator and rotor have lagging power factors, the coil sides which will have maximum current will be different. These are then identified and then positions of $\bar{B}_s$ and $\bar{B}_r$ are positioned so that $\bar{B}_{\text{net}} = \bar{B}_s + \bar{B}_r$ as shown in figure 5(b).